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A Sufficient Synchronization Criterion for Memristively Coupled FitzHugh-Nagumo Oscillators

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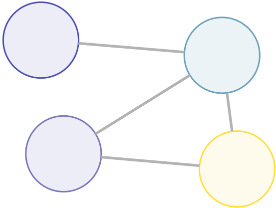
Introduction

Oscillator networks

... are relevant for many fields like

- Neural networks
- Power systems
- Neuromorphic computing



Understanding

... synchronization is a key part of designing novel computing hardware

Promising

... architectures can compute e.g. the traveling salesman problem more efficiently than today's computers

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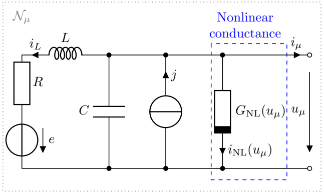
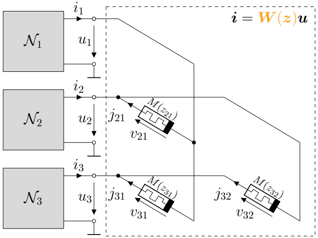
System

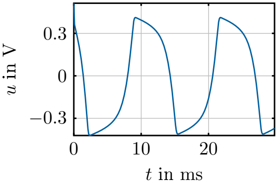
FitzHugh-Nagumo Oscillator (FNO)

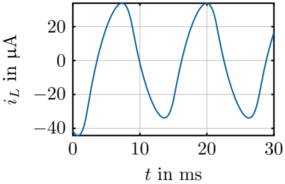


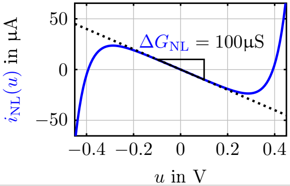
Topology



Oscillation and Nonlinearity







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Argument

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Synchronization

Oscillators are synchronized if the states are identical:

- $\boldsymbol{u} = \hat{u}\mathbb{1}$
- $\boldsymbol{i}_L = \hat{\imath}\mathbb{1}$

Projection matrix $\boldsymbol{P}$ extracts unsynchronized parts:

- $\boldsymbol{P}\mathbb{1} = 0$
- $\boldsymbol{P}\boldsymbol{v} = \boldsymbol{v}$ if $\mathbb{1}^\mathrm{T}\boldsymbol{v} = 0$

Lyapunov Candidate

$$V = \frac{1}{2}\begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{i}_L\end{bmatrix}^\mathrm{T}\begin{bmatrix} C\boldsymbol{P} & 0 \\ 0 & L\boldsymbol{P}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{i}_L\end{bmatrix}$$

V should be

- zero when the oscillators are synchronized
- decreasing when they are not

Derivation

$$\dot{V} = -\boldsymbol{i}_L^\mathrm{T}\boldsymbol{P}R\boldsymbol{i}_L \underbrace{-\boldsymbol{u}^\mathrm{T}\boldsymbol{P}\boldsymbol{W}(\boldsymbol{z})\boldsymbol{u}}_{p_\text{coupling}} \underbrace{-\boldsymbol{u}^\mathrm{T}\boldsymbol{P}\boldsymbol{i}_\mathrm{NL}(\boldsymbol{u})}_{p_\mathrm{NL} \leq \boldsymbol{u}^\mathrm{T}\boldsymbol{P}\Delta G_\mathrm{NL}\boldsymbol{u}} \leq -\boldsymbol{i}_L^\mathrm{T}\boldsymbol{P}R\boldsymbol{i}_L - \boldsymbol{u}^\mathrm{T}\boldsymbol{P}[\boldsymbol{W}(\boldsymbol{z}) - \Delta G_\mathrm{NL}\mathbf{1}]\boldsymbol{u} \overset{!}{\leq} 0$$

A sufficient synchronization criterion is $\lambda_2\{\boldsymbol{W}(\boldsymbol{z})\} > \max\{0, \Delta G_\mathrm{NL}\}$.

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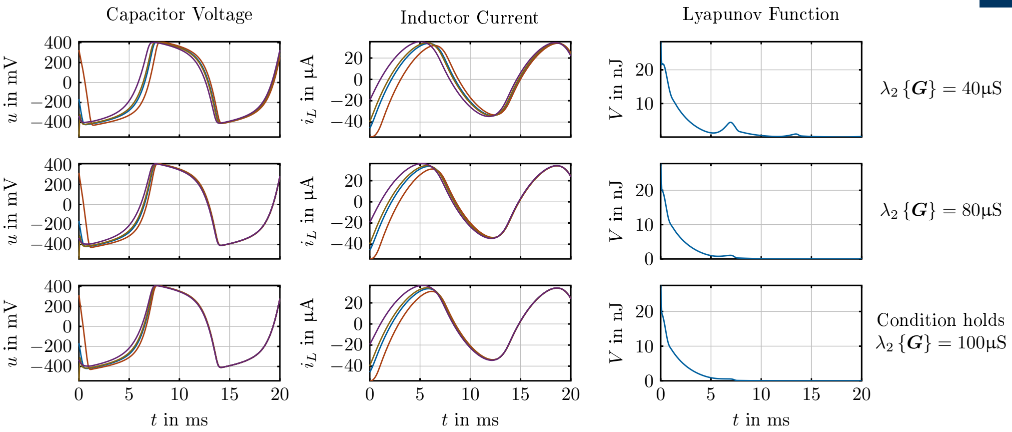
Synchronization of four statically coupled FNOs



Synchronization beyond condition with non-monotonous Lyapunov function.

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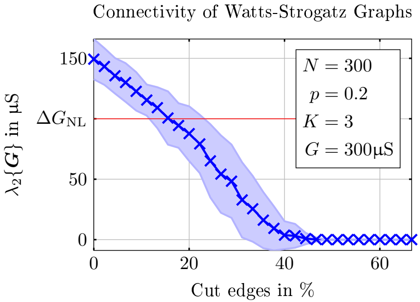
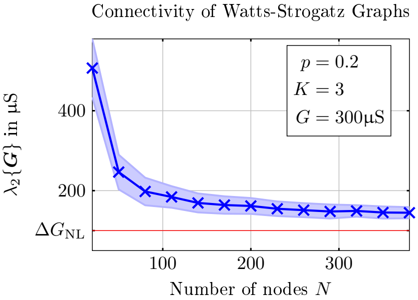
Algebraic connectivity of Random Graphs

Cutting Edges



Connectivity drops linearly with memristors switching to high resistive state.

Varying Node Numbers



Changing the number of oscillators has limited effect on synchronization.

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Conclusion

Key factors for the synchronization criterion are

- Algebraic connectivity of the coupling graph
- Maximal magnitude of negative nonlinearity slope

For Small-World Graphs

- Memristors switching to high resistive state have linear impact on connectivity
- Synchronization behavior will likely change slowly when disconnecting oscillators

For more detail see [1].

Sources

[1] Robin Lautenbacher et al. "Sufficient Synchronization Conditions for Resistively and Memristively Coupled Oscillators of FitzHugh-Nagumo-type." In: *Discover Applied Sciences* (2024). in revision.